Title: Making Connections: Matrices, Graphs, and Linear Systems

Brief Overview:

Students will use graphing calculators to solve systems of linear equations in two ways. They will first solve the systems by graphing the equations and finding the point of intersection. They will then solve systems of equations by writing related matrices and solving by using inverse matrices.

Link to Standards:

- **Problem Solving** Students will demonstrate their ability to solve real-life problems through the use of matrices and graphing calculators.
- **Connections** Students will learn two different methods of solving systems of equations - solving by graphing and solving by matrices.
- **Discrete Math** Students will perform matrix operations by hand and on the calculator to solve problems.

Grade/Level:

Grades 9-12; Algebra I, Algebra II

Duration/Length:

This lesson will take 4 or 5 periods (45 min.).

Prerequisite Knowledge:

Students should be able to do the following:

- Solve a system of linear equations algebraically
- Graph a system of linear equations on graph paper
- Given a real-world problem, write a system of equations

Objectives:

Students will be able to:

- graph a system of equations on the graphing calculator and locate the point of intersection.
- solve a system of linear equations using matrices, first by paper and pencil, then on the graphing calculator.
- solve a system of linear equations using matrices on the graphing calculator.
- solve a system of equations representing a real-world problem.

Materials/Resources/Printed Materials:

- TI-82 or TI-83 Graphing Calculators
- Student Activity Worksheets # 1-4
- TI-82 Overhead Projector

Development/Procedures:

Day 1:

- Teacher will demonstrate graphing a pair of lines and finding the intersection point on the overhead graphing calculator. (See teacher notes.)

- Students will work with a partner on Activity Worksheet 1 to practice solving a system of equations by graphing.

Day 2:

- Teacher will provide instruction on working with matrices using paper and pencil. The following should be discussed: dimensions of a matrix, multiplying 2X2 matrices, the identity matrix, and finding the determinant and inverse of a 2X2 matrix. (See teacher notes.)

Day 3:

- Students will work in pairs on Activity Worksheet 2 on exploring matrices on the graphing calculator. (Note: Teacher will provide assistance, as needed, on the Guided Practice. Students can complete the Independent Practice in class or as homework.)

Day 4:

- Teacher will demonstrate the complete solution of a linear system of two equations, two unknowns using matrices. (See teacher note.)

- Students will work with a partner on Activity Worksheet 3 to practice solving a system of equations by using matrices.

Day 6:

- Students will work in pairs on Activity Worksheet 4 on solving real-world problems that can be represented by a system of equations.

Evaluation:

Student achievement will be measured by checking activity worksheets. The teacher will also be able to judge student success by circulating around the room and by serving as a resource for individual students.

Extension/Follow Up:

In future lessons, explore further use of matrices to solve real-world problems, including communication networks, digraphs, and various discrete math topics.

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TEACHER NOTES

DAY 1

Solve systems of linear equations graphically on the TI - 82.
Make sure to have the students check their modes to see if they are set on rectangular. Also, they may have a split screen (MODE) or axes off (Format) due to some program or previous setting. These are problems needing correction in advance.

They must solve equations for y in terms of x. Begin by using the standard graphing window (ZOOM standard), then go over (in ex 3) how to adjust the window to see the point of intersection. Graph the following systems on the overhead graphing calculator:

1. $y = 2x + 5$ and $y = -x + 8$

2. $5x - 5y = 16$ and $x - 2y = 4$

3. $x + y = 14$ and $x - y = 10$

Solutions: (1,7) (2.4, -.8) (12, 2)

Pair up the students and have them work on Activity I.

DAY 2

Give examples of matrices of different dimensions (row x column), such as a 2 X 3 , a 3 X 2, several square matrices, a 3 x 5, a 1 x 4, and a 3 x 1.

To multiply two matrices, they must be of compatible dimensions (i.e., the number of columns of the first matrix must equal the number of rows in the second matrix). The dimensions of the answer to A (B), if A is an m x n matrix and B is an n x p matrix is an m x p matrix.

Example of multiplication:

If $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -3 \\ -1 & 4 \end{bmatrix}$

then $A(B) = \begin{bmatrix} 2(0) + 3(-1) & 2(-3) + 3(4) \\ 1(0) + 5(-1) & 1(-3) + 5(4) \end{bmatrix}$ or $\begin{bmatrix} -3 & 6 \\ -5 & 17 \end{bmatrix}$

NOTE THAT MATRIX MULTIPLICATION IS NOT COMMUTATIVE!

THAT IS: A(B) generally does not equal B (A)

If $C = \begin{bmatrix} -5 & 7 \\ -2 & 0 \end{bmatrix}$ then do the following multiplications.

1. B(C) 2. C(A) 3. C(B) 4. B(A)

The identity matrix is any square matrix with the top-left-to-bottom-right diagonal elements equal to ones, and all the other entries equal to zeros.

If I is the identity matrix of appropriate size, then I (A) = A(I) = A

The identity matrix for a 2 X 2 matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The determinant of a 2 X 2 matrix is easily found. It is a scalar number, not a matrix.

For matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the determinant is a(d) - b(c)

The determinant is denoted $|A|$

(similar to absolute value symbols)

*Now to find the inverse of matrix A

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The inverse can be used to solve a system of linear systems in the following way:

If A is the coefficient matrix and X is the variable matrix and C is the matrix is the constant matrix for a system of equations written in standard form, then

$$A\ (X) = C$$

So $A^{-1}\ A\ (X) = A^{-1}\ C$ (left-hand multiplication on both sides!)

Or $I\ (X) = A^{-1}\ C$

and the solution will be given by:

$$X = A^{-1}\ C$$

Day 3

The worksheet on solving systems using matrices on the graphing calculator is fairly self-explanatory. Make sure they fill in the matrices on the worksheet as an explanation of what they are doing.

Day 4

Demonstrate solving the following systems using matrices, writing the standard form, setting up the required matrices, and performing the matrix algebra on the overhead calculator.

$5x + 4y = 6$	$y = 2x - 3$
$-2x - 3y = -1$	$4x - 2y = 11$
Solutions: (2,-1)	no solution (show what happens)

$5x + 16y = 15$

$4y + 2x = 1$ Solution:(-3.66667, 2.08333)

Activity Worksheet 3 has three problems for them to do on their own or with a partner, and may be finished for homework if necessary.

Day 5

Activity Worksheet 4 has several real-world problems for which students will have to write their own equations, and solve according to given directions, either by graphing or matrices. All questions should be fully answered.

Activity #1

Solving A Linear System by Graphing on The Graphing Calculator

Group Members

Reader/Recorder_____

Calculator Operator _____

I. Guided Practice

The system of equations: $y = x - 6$

$y = -2x$ will intersect.

Why?_____

This intersection point will be the "solution" to the system of equations.

* To enter the equations into the calculator:

Enter [Y=] (Clear out any existing equations)

Type in equation 1 (y_1) Press [ENTER]

Type in equation 2 (y_2) Press [ENTER]

Before pressing [**GRAPH**] , press [ZOOM] 6 to set a standard graphing window.

Now press GRAPH. Can you see the point of intersection? If not adjust the viewing window for the desired minimum and maximum values of x and y, or you can TRACE and ZOOM in.

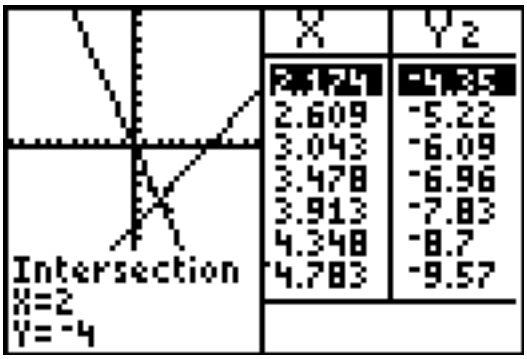
* To find the point of intersection

Press 2nd TRACE (calc) 5 ENTER

Guess (by moving the cursor) the closest point on the first curve. ENTER

Guess (by moving the cursor) the closest point on the second curve. ENTER

Final Guess? ENTER

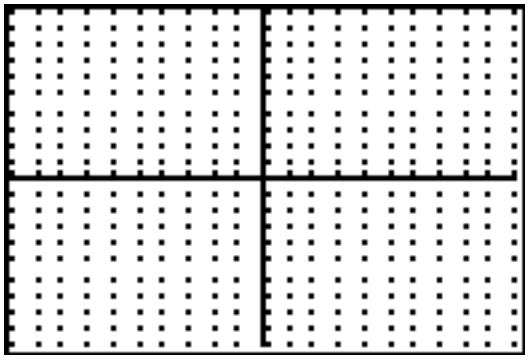


II. Independent Practice

Graph and sketch each system of equations. Now find the point of intersection (if it exists). ZOOM or adjust your WINDOW if this point is not visible. Use the intersect command to find the coordinates of the intersection point.

1. $y = x + 3$

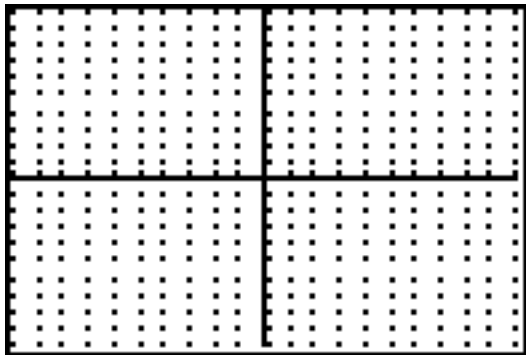
   $y = 2x + 8$



Solution (,)

2. $y = 4x$

   $y = -3x$

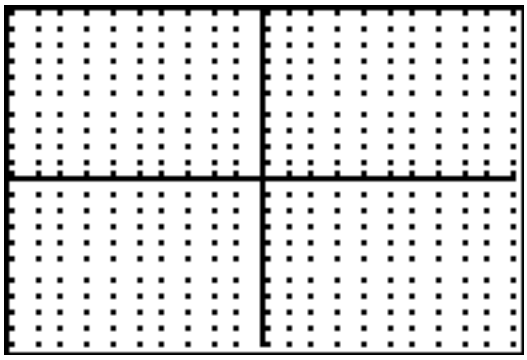


Solution (,)

3. $y = 3x - 9$

   $6x - 2y = 10$

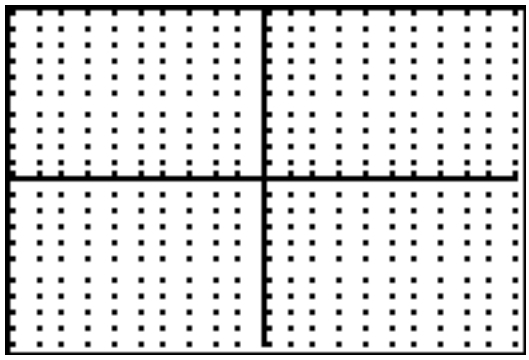
(Remember to solve for y !)



Solution (,)

4. $y = 4x + 7$

   $3x + 2y = 30$



Solution (,)

Activity #2

Exploring Matrices on the Graphing Calculator

Group Members

Reader/Recorder_____

Calculator Operator_____

I. Guided Practice

You have used pencil and paper to work with matrices. Now let's try using the graphing calculator. Work through the following examples.

Example 1)

* To enter a matrix into your calculator:

Define matrix A = $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ in your calculator.

Enter: [MATRX] . Select EDIT . Press 1

Press 2 [ENTER] 2 [ENTER] (this enters the matrix dimension)

3 [ENTER] -1 [ENTER]

4 [ENTER] 2 [ENTER] (this enters the entire matrix)

* To display the matrix:

Press [2nd] [QUIT]

Then [MATRX] 1 Does it look the way you want it to?

Yes/ no? _____

[CLEAR]

* Find the determinant:

Enter : [MATRX] Select MATH. Press 1.

Then [MATRX] 1 [)] [ENTER]

What is the determinant? _____

[CLEAR]

* Find the inverse:

Enter: [MATRX] 1 [X $^{-1}$]

[ENTER] What is the inverse?

$A^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$

Example 2)

Enter matrix B = $\begin{bmatrix} 3 & 1 \\ 4 & -2 \end{bmatrix}$

Find AB

Enter : [MATRX] 1 [MATRX] 2 [ENTER]

$AB = \begin{bmatrix} & \\ & \end{bmatrix}$

Now find BA

BA = [ ]

Is matrix multiplication commutative? Yes/No_____

* Find A^{-1}

Enter: [MATRX] 1 [X^{-1}] [ENTER]

A^{-1} = [ ]

* Find A^{-1} B

A^{-1} B = [ ]

* To clear these matrices:

Enter : [2nd] MEM . Choose 2 [DELETE.] Then, choose 5 Matrix

[ENTER] [ENTER]

II. Independent Practice

$$A = \begin{bmatrix} 8 & 6 \\ 9 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 \\ 8 & -4 \\ -2 & 6 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & -3 & 6 \\ 5 & 4 & -2 \end{bmatrix}$$

Find the following:

1. BC

2. BA

3. A^{-1}

4. A^{-1} C

5. A^{-1} A

Activity #3

Practicing Solving Systems With Matrices

Name_____

_____

Solve each system, step by step with matrices, as illustrated on # 1.

Solve:

1. $3x - 4y = 2$

   $5x + 6 = 16$

2. $2x + 4y = 16$

   $4x - 8y = -20$

Coefficient Matrix x Variable Matrix = Constant Matrix

$$\begin{bmatrix} & \\ & \end{bmatrix} \quad \begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

A (X) = B

so, X = A^{-1} B

3. $x - y = .6$

   $y + 4x - 3 = 17.4$ (Remember to write equations in **standard** form.)

Activity #4

Solving Real-World Problems

Name _____

_____

You now have two methods of solving systems of equations using your graphing calculator--by graphing or by using matrices. Let's apply these methods in solving some real-world problems!

I. Karen has a total of $4000, part in a savings account and the rest in certificate of deposit (a certificate). Her savings account earns 6.5% interest annually. The CD pays 8% if the money is invested for 1 year. How much does she have in each investment if her interest earnings for the year will be $297.50?

1. Write a system of equations to describe this situation.

2. Write the related matrix equation.

A * X = B

3. Solve using your graphing calculator. What are your results?

4. How much does Karen have in each investment?

_____

5. Suppose that the situation were changed so that Karen's savings account earned 7% instead of 6.5%. How much would she have in each investment if she still earns $297.50 interest? (You should only have to make one change in your matrix!)

_____

6. Suppose the savings account earns 6.5% interest and the CD earns 8%. If Karen wants to increase her interest earnings to $310, determine how she should invest her $4000. How much should she invest in the savings account? How much should she invest in the CD?

_____

II. Sara has $300 in her savings account and withdraws $8 per week. Sam has $125 in his savings account and deposits $15 per week.

1. Write an equation describing the amount of money (y) that Sara has in her account at the end of a given week. _____

2. Write an equation describing the amount of money that Sam has in his account at the end of a given week. _____

3. Graph this system of equations on your graphing calculator.

   Next find a window setting that shows the point of intersection. You may have to try a few before you can obtain a viewing window that shows the portion of the graph that you want to see.

   Show your window setting.

   $x_{min} =$ ____
   $x_{max} =$ ____
   $x_{scl} =$ ____
   $y_{min} =$ ____
   $y_{max} =$ ____
   $y_{scl} =$ ____
   $x_{res} =$ ____

   What is the point of intersection? _____

4. After how many weeks will Sara and Sam have the same amount of money? _____

5. How much will they each have? _____

III. Suppose that Amelia flies her small plane 80 miles from Tampa to Orlando in 40 minutes (2/3 hour) against the wind. She returned to Tampa in 32 minutes (8/15 hour) with the wind against her back.

1. From Tampa to Orlando, what was her rate of speed? _____miles/hour

2. From Orlando to Tampa, what was her rate of speed? _____miles/hour

3. Let x be the average speed of the plane without wind and y be the speed of the wind. Write a system of equations to describe the situation.

4. Solve this system using graphing or matrices. What was the average speed of the plane? of the wind?

_____

_____

5. Suppose it took 50 minutes to fly to Orlando against the wind and 40 minutes for the return flight with the wind. Find the speed of the plane and the speed of the wind in miles per hour under these conditions.

_____

_____